ERWIN SCHRÖDINGER’S ALARMING PHENOMENON

\[ -\frac{\hbar^2}{2m} \nabla^2 \Psi + V \Psi = i\hbar \frac{\partial \Psi}{\partial t} \]

The Proper Vibrations of the Expanding Universe

Physica VI, 899 (1939)

Rocky Kolb
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November 9, 2016
Arizona State University
ERWIN SCHröDINGER’S ALARMING PHENOMENON

Schrödinger’s 1939 deep insight into particle creation in the expanding universe
- was correct in principle (although some technical missteps)
- was not then (and isn’t much now) appreciated
- was alarming to Schrödinger, but now a fundamental part of cosmology
- has profound implication for understanding our present universe
  - we are all amplified quantum fluctuations!
  - gravitons
  - dark matter?
  - other?

Lessons to be learned
- lower the anchor of our peaceful studies into the ground of eternity
- don’t be alarmed
- trust your equations
- beautiful physics has far-reaching implications
THE PROPER VIBRATIONS OF THE EXPANDING UNIVERSE
by ERWIN SCHRÖDINGER
Physica VI, 899 (1939)

§ 1. Introduction and summary. Wave mechanics imposes an a priori reason for assuming space to be closed; for then and only then are its proper modes discontinuous and provide an adequate description of the observed atomicity of matter and light. — Einstein's theory of gravitation imposes an a priori reason for assuming space to be, if closed, expanding or contracting; for this theory does not admit of a stable static solution. — The observed facts are, to say the least, not contrary to these assumptions.

This makes it imperative to generalize to expanding (or contracting) universes the investigation of proper vibrations, started for the static cases (Einstein- and de Sitter-universe) by the present writer and two of his collaborators 1). The task is an easy one.

The broad results are largely (in part even entirely) independent of the time-law of expansion. In the cases of main practical interest, i.e. with the present slow time rate of expansion and with wave lengths small compared with the radius of curvature of space (R), they are the following.

For light: when referred to the customary co-moving coordinates, an arbitrary wave process exhibits essentially the same succession of states as without expansion. Briefly, the wave function shares the general dilatation. Hence all wave lengths increase proportionally to the radius of curvature. — The time rate of events is slowed down. It is, in every moment, proportional to $R^{-1}$. Moreover all intensities are affected by a common factor such as to make the total energy of an arbitrary wave process proportional to $R^{-1}$.

For the material particle the broad results are these: a strictly monochromatic process (i.e. a proper vibration) again shares the
common dilatation, so that its wave length $\lambda$ is proportional to $R$, as before. From the changing $\lambda$ the changing frequency is calculated by de Broglie's formula. This implies different frequencies to be affected by different factors. Therefore an arbitrary wave function can no longer be said to simply share the common dilatation. But since de Broglie's dispersion formula persists, the familiar connection (momentum = $\hbar/\lambda$) between linear group velocity (= particle velocity) and wave length is also preserved, which causes the former or more precisely the momentum, to decrease proportional to $R^{-1}$.

As regards the amplitudes, the most reliable information about them, valid for any particle wave function whatsoever, is this, that the normalisation is rigorously conserved during the expansion.

These are the broad results. A finer and particularly interesting phenomenon is the following.

The decomposition of an arbitrary wave function into proper vibrations is rigorous, as far as the functions of space (amplitude-functions) are concerned, which, by the-way, are exactly the same as in the static universe. But it is known, that, with the latter, two frequencies, equal but of opposite sign, belong to every space function. These two proper vibrations cannot be rigorously separated in the expanding universe. That means to say, that if in a certain moment only one of them is present, the other one can turn up in the course of time.

Generally speaking this is a phenomenon of outstanding importance. With particles it would mean production or annihilation of matter, merely by the expansion, whereas with light there would be a production of light travelling in the opposite direction, thus a sort of reflexion of light in homogeneous space. Alarmed by these prospects, I have investigated the question in more detail. Fortunately the equations admit of a solution by familiar functions, if $R$ is a linear function of time. It turns out, that in this case the alarming phenomena do not occur, even within arbitrarily long periods of time.

Waves travelling in one direction can be rigorously separated from those travelling in the opposite direction. The results for D'Alambert's equation (light) and Gordon's equation (material particles), which have been used throughout in this paper for the sake of simplicity, are given in sect. 5 and 6 respectively. I have confirmed the results with Dirac's equation, but reserve it to a subsequent paper.

Even in an expanding universe, a particle's wavefunction can be decomposed into "proper vibrations" (positive & negative frequency modes):

$$\Psi(t) = \frac{a}{\sqrt{2\omega}} e^{-i\omega t} + \frac{\beta}{\sqrt{2\omega}} e^{+i\omega t}$$

Particle occupancy number $\propto |\beta|^2$.

If start with pure incoming or outgoing waves, in and out will become mixed.

The expansion of the universe creates particles!

This alarms me [ed. why?], so I wrote a paper.
Schrödinger’s two favorite phrases:

1. alarming phenomenon
2. mutual adulteration

Schrödinger was alarmed by creation of a single particle

per Hubble time \((H_0^{-1} \sim 10^{10}\text{ yr})\)

per Hubble volume \((H_0^{-3} \sim 10^{57}\text{ km}^3)\)

with Hubble energy \((H_0 \sim 10^{-33}\text{eV})\)
His private life seemed strange to bourgeois people like ourselves. But all this does not matter. He was a most lovable person, independent, amusing, temperamental, kind and generous, and he had a most perfect and efficient brain.

Max Born on Schrödinger's “private life”
Schrödinger’s Alarming Times

1926: “Quantisierung als Eigenwertproblem,” Annalen der Physik. 384, 273

1927: Schrödinger visits U.S.
   Found noise and dirt of New York “shattering”
   Found Chicago worse, feared “bandits who spring with loaded guns from speeding autos.” (Anny liked Chicago.)
Schrödinger departs UZH for Berlin.


1936: Schrödinger departs Oxford for Graz in a miscalculation of the political situation that was, in his words, an “unprecedented stupidity.”

1938: 12 March, Anschluss; 26 August, Schrödinger dismissed; 14 September, Erwin & Anny left Graz for Rome with ten Marks, three suitcases, sans Nobel medal; met in Rome by Fermi; asylum in the Vatican.

Biographical info. from Walter Moore, Schrödinger, Life and Thought (Cambridge Univ. Press, 1992)
Dublin  
February 10, 1951  

Dear Fermi,  

..... I beg you to help me remove once and for all, a remorse that I cannot help associating with my memory of you at our last meeting, namely that I still owe you Lire 400 val. Sept 1938. To re-calculate this sum to date, now that all money-value has gone down is very difficult, but I think something like 200 Swedish Crowns would be a modest estimate for re-payment. If you agree and if you still have an account at Stockholm, this would be very simple. If the later is not the case, please indicate me your bankers’ account at Chicago, and I hope to manage even so.  

.....  

Yours very sincerely,  

E. Schrödinger
Chicago
February 27, 1951

Dear Schrödinger [sic],

..... As to the old debt that you mention, I believe that you are estimating the value of 400 lire too high. At that time the lire was worth about one twentieth of one dollar and it seems therefore a $20.00 settlement would be correct. I no longer have an account in Sweden. My bank here in Chicago is the University National Bank, 1354 East 55th Street, Chicago 15. Please however, be sure if there are any difficulties whatsoever about transferring this amount not to worry about it because it is certainly not worth it.

.....

Yours very sincerely,
Enrico Fermi
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Schrödinger the Cosmologist

In Belgium Schrödinger met cosmologist Abbé Georges Lemaître.

Previously several interactions with Sir Arthur Stanley Eddington.

July 1939 Nature of the Nebular Red-Shift

August 1939 The Proper Vibrations of the Expanding Universe

1956 Expanding Universes, Cambridge Univ. Press
Expansion of the Universe

Hubble's discovery paper
*Proc. Natl. Acad. Sci. 15*, 168 (1929)
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1939: October, Schrödinger departs Belgium for Dublin.

Biographical info. from Walter Moore, *Schrödinger, Life and Thought* (Cambridge Univ. Press, 1992)
Kepler’s Alarming Times

When the storm rages and the state is threatened by shipwreck, we can do nothing more noble than to lower the anchor of our peaceful studies into the ground of eternity.

– Johannes Kepler
Schrödinger’s Alarming Phenomenon

Why was Schrödinger alarmed?

How to understand mutual adulteration (particle creation)?

Why is it important?
Schrödinger’s Alarming Phenomenon

Why was Schrödinger alarmed?

- Appearance of particles from the vacuum sounds crazy.
- Technical issues with calculation:
  - Only create particles with mass less than expansion rate $H$ (today $H_0 \sim 10^{-33}$ eV).
  - Only create particles if violate Weyl Conformal Invariance (don’t create photons).
  - Would Schrödinger still have been alarmed?
- Schrödinger looked for (and found) a cosmological solution without mutual adulteration (not a very physical solution).
- Perhaps he thought it was conceptual challenge to Quantum Mechanics, Quantum Field Theory, or General Relativity.
- Infinite particle creation in standard big-bang at $t = 0$.
- (Sometimes should just follow the equations).
Schrödinger’s Alarming Phenomenon

How to understand mutual adulteration (particle creation)?

The Quantum Vacuum
Derek Leinweber, University of Adelaide
Schrödinger’s Alarming Phenomenon

How to understand mutual adulteration (particle creation)?

The Quantum Vacuum

External Fields Can Disturb The Quantum Vacuum
Disturbing the Quantum Vacuum

Changing Electric Field → Particle creation

\[ e^+ \rightarrow e^- \]

Particle creation if energy gained in acceleration from electric field over a Compton wavelength exceeds the particle’s rest mass.

Heisenberg & Euler (1935); Weisskopf (1936); Schwinger (1951)
Disturbing the Quantum Vacuum

Tidal gravitational field → Particle creation

$e^+ \quad e^-$

Black Hole

$e^+ \quad e^-$

tidal gravity field

Particle creation if energy gained in acceleration from gravitational field over a Compton wavelength exceeds the particle’s rest mass.

Hawking (1974); Bekenstein (1972)
Disturbing the Quantum Vacuum

Expanding universe → Particle creation

Particle creation if energy gained in expansion over a Compton wavelength exceeds the particle’s rest mass.

Schrödinger’s Alarming Phenomenon (1939)
Schrödinger’s Alarming Phenomenon

Technical Details:

Quantum fields in curved space

N.D. Birrell & P.C.W. Davies

Cambridge Monographs on Mathematical Physics
Schrödinger’s Alarming Phenomenon

Free quantum scalar field $\chi$ of mass $M$ in Minkowski space:

$$S = \int d^4x \left[ \frac{1}{2} \partial^\mu \chi \partial_\mu \chi - \frac{1}{2} M^2 \chi^2 \right]$$

Mode expand and make plane-wave ansatz:

$$\chi(x, t) = \sum_k \hat{a}_k e^{ik \cdot x} \chi_k(t) + \hat{a}_k^\dagger e^{-ik \cdot x} \chi_k^*(t)$$

Equation of motion (Klein-Gordon equation):

$$\ddot{\chi}_k(t) + \omega_k^2 \chi_k(t) = 0 \quad \omega_k^2 = \left| k \right|^2 + M^2$$

Choose pure outgoing (+ frequency) solution

$$\chi_k(t) = \frac{1}{\sqrt{2 \omega_k}} e^{-i\omega_k t}$$

other solution: $$\chi_k(t) = \frac{1}{\sqrt{2 \omega_k}} e^{+i\omega_k t}$$
Schrödinger’s Alarming Phenomenon

Couple scalar field $\chi$ to gravity:

$$S = \int d^4 x \sqrt{-g} \left[ \frac{1}{2} g^{\mu\nu} \partial_\mu \chi \partial_\nu \chi - \frac{1}{2} M^2 \chi^2 - \xi R \chi^2 \right]$$

$g_{\mu\nu}$ is the metric tensor

Cosmological background, metric one function, scale factor $a(t)$

Hubble parameter, $\dot{a}(t)/a(t) = H(t)$ expansion rate of the universe

Equation of motion:

$$\ddot{\chi}_k(t) + \frac{\dot{a}(t)}{a(t)} \dot{\chi}_k(t) + \left[ \frac{1}{a^2(t)} \left| \vec{k} \right|^2 - (1 - 6\xi) \left( \frac{\dot{a}^2(t)}{a^2(t)} + \frac{\ddot{a}(t)}{a(t)} \right) \right] + M^2 \chi_k(t) = 0$$

If express in terms of conformal time $\eta$, $d\eta \equiv a^{-1}(t) \, dt$:

$$\chi_k''(\eta) + \omega_k^2(\eta) \chi_k(\eta) = 0 \quad \omega_k^2(\eta) = \left| \vec{k} \right|^2 - (1 - 6\xi) \frac{a''(\eta)}{a(\eta)} + a^2(\eta) M^2$$

wave equation with time-dependent mass that depends on evolution in time of the scale factor
Schrödinger’s Alarming Phenomenon

Solutions to wave equation are adulterated:

\[ \chi''_k(\eta) + \omega^2_k(\eta) \chi_k(\eta) = 0 \quad \omega^2_k(\eta) = \left| \vec{k} \right|^2 - (1 - 6\xi) \frac{a''(\eta)}{a(\eta)} + a^2(\eta)M^2 \]

Pure outgoing (+ frequency) solution \[ \chi_k(\eta) = \frac{1}{\sqrt{2\omega_k(\eta)}} e^{-i \int \omega_k(\eta) d\eta} \]

is a good solution if \[ \left| \frac{\omega'_k(\eta)}{\omega^2_k(\eta)} \right|^2 \ll 1 \quad \text{and} \quad \left| \frac{\omega''_k(\eta)}{\omega^3_k(\eta)} \right|^2 \ll 1 \]

Abrupt changes in \( a(\eta) \) leads to abrupt changes in \( \omega_k(\eta) \), which adulterates positive and negative frequency modes, leading to Schrödinger’s Alarming Phenomenon of particle creation in the expanding universe.
Schrödinger’s Alarming Phenomenon

Solutions to wave equation include both $+$ and $-$ frequency terms

\[
\chi_k(\eta) = \frac{\alpha_k(\eta)}{\sqrt{2\omega_k(\eta)}} e^{-i \int \omega_k(\eta) d\eta} + \frac{\beta_k(\eta)}{\sqrt{2\omega_k(\eta)}} e^{+i \int \omega_k(\eta) d\eta}
\]

If start with only outgoing waves, $\beta_k(\eta) = 0$, generate incoming waves, $\beta_k(\eta) \neq 0$.

Comoving number density of particles at late time is

\[
n = \frac{1}{(2\pi)^3} \int d^3\vec{k} \left| \beta_k(\eta) \right|^2
\]
Schrödinger’s Alarming Phenomenon

- Expansion of the Universe leads to time dependence of the coupling of fields to gravity

- Expansion of the universe leads to creation of all species of particles so long as there is a “time” dependence to $\omega_k(\eta)$

$$\omega_k^2(\eta) = \left| \vec{k} \right|^2 - (1 - 6\xi) \frac{a''(\eta)}{a(\eta)} + a^2(\eta) M^2$$

Note: $\frac{a''(\eta)}{a(\eta)}$ can be positive.

- Adulteration efficacy depends on abruptness of change in $\omega_k(\eta)$

$$\left| \frac{\omega'_k(\eta)}{\omega^2_k(\eta)} \right|^2 \quad \text{and} \quad \left| \frac{\omega''_k(\eta)}{\omega^3_k(\eta)} \right|^2$$

Production suppressed if $M \gtrsim H$

Particles “produced” when $\left| \vec{k} \right| / a \simeq H$ ($\xi = 0$)

- Many subtleties glossed over
Schrödinger’s Alarming Phenomenon

The graph illustrates the transition between adiabatic and non-adiabatic regimes. The parameter $\beta_k$ is plotted against $\eta$. The graph shows a sharp transition from adiabatic to non-adiabatic behavior, with oscillations in the non-adiabatic region.
Schrödinger’s Alarming Phenomenon

Why is it important?
If you can look into the seeds of time
And say which grain will grow and which will not,
Speak then to me, who neither beg nor fear
Your favours nor your hate.

— Macbeth (Banquo)
Primordial Seeds of Structure

CBR: snapshot of the universe 380,000 AB

Correlations on scales $\gg 380,000$ light years
More Than 380,000 Light Years In Less Than 380,000 Years?

- \( v \leq c \) for velocity through space
- no limit on expansion velocity of space
- “acausal” requires “accelerated” expansion
Non-Inflationary Cosmology

$\lambda \propto a$

$R_H \propto \begin{cases} a^{3/2} & \text{(Matter)} \\ a^2 & \text{(Radiation)} \end{cases}$

$R_H = H^{-1}$

$\lambda < R_H$

$\lambda > R_H$

380,000 yr AB

today
Inflationary Cosmology

\[ \lambda \propto a \quad R_H \propto \begin{cases} a^{3/2} & \text{(Matter)} \\ a^2 & \text{(Radiation)} \\ a^0 & \text{(Inflation)} \end{cases} \]

\[
\left( \frac{\lambda}{R_H} \right) \propto \left( \frac{a}{a/\dot{a}} \right) = \ddot{a} > 0
\]

380,000 yr AB

accelerated phase
(inflation)
Inflation

- Scalar field has potential energy $V(\phi)$ and kinetic energy $\dot{\phi}^2$
- Potential energy (zero-momentum mode of $\phi$) dominates $\Rightarrow$ acceleration
- Schrödinger’s Alarming Phenomenon produces inflaton particles ($\hbar \neq 0$)
- Particles produced with non-zero momentum $\delta \phi \to \delta \rho \to \delta T$
- Particles produced when they cross Hubble radius during inflation fluctuations on all scales with approximately same amplitude (Harrison—Zel’dovich)
A pattern of vacuum quantum fluctuations
\( \hbar \rightarrow 0 \)

You are an amplified quantum fluctuation!
Quantum fluctuations, once microscopic, have been stretched ...

... to be as large as the observable universe!

- The map of CMB $\Delta T/T$ is a map of quantum fluctuations
- produced $10^{-35}$ seconds after the bang during primordial inflation
- when the universe was dominated by vacuum energy
- and rapid expansion ripped particles out of the quantum vacuum
- (Schrödinger's alarming phenomenon)
- producing primordial seeds of structure that grew to all we see
- (you are an amplified quantum fluctuation),
- and encoded in the pattern is the imprint of fundamental physics.
Disturbing the Quantum Vacuum

Expanding Universe $\rightarrow$ Particle creation

Particle creation if energy gained in expansion over a Compton wavelength exceeds the particle’s rest mass.

Schrödinger’s Alarming Phenomenon applies to gravitons
Inflationary Cosmology

\[ R_H = H^{-1} \]

\[ \delta \phi \Rightarrow \]
- Temp. fluctuations
- Seeds of structure
- Gravitational waves

\[ \lambda < R_H \]

\[ \lambda > R_H \]

\[ \delta \phi : \text{"Schrödinger's Alarming Phenomenon"} \]
Schroedinger’s Alarming Phenomenon Leaves Imprint of Inflation

- Inflation: Big Bang plus $10^{-35}$ seconds
- CMB fluctuations (density perturbations, gravitational waves)
- Big Bang plus 380,000 Years
- Big Bang plus 14 Billion Years
WIMPzillas
Chung, Kolb, Riotto; Kuzmin, Tkachev; .... Kolb & Long

• Schrödinger’s Alarming Phenomenon applies to all particles*
• All particles* produced during inflation
• Could dark matter be produced during inflation?

- Inflation signifies a new mass scale.
- $H_e$, expansion rate at end of inflation, comparable to inflaton mass.
- Might expect other particles with mass comparable to inflaton mass.
- If one of them stable, natural candidate for dark matter (WIMPzilla miracle).

* So long as conformal symmetry violated.
Schroedinger’s Alarming Phenomenon Connects the Quantum & the Cosmos

When we try to pick out anything by itself, we find it bound fast, by a thousand invisible cords that cannot be broken, to everything else in the universe.

– John Muir
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